

Hand-in @ lecture #4

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2026-03-26

1.1.15

$(\neg q \wedge (p \rightarrow p)) \rightarrow \neg q$ is a tautology iff its negation has any solution:

$$(\neg q \wedge (p \rightarrow q)) \rightarrow \neg q$$

1. $\neg((\neg q \wedge (p \rightarrow q)) \rightarrow \neg q) \vee$ Negation
 2. $(\neg q \wedge (p \rightarrow q)) \wedge \neg \neg q \vee$ Material nonimplication
 3. $(\neg q \wedge (p \rightarrow q)) \wedge q \vee$ Double negation elimination
 4. $\neg q \wedge (p \rightarrow q) \wedge q$ Associative conjunction simplification
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The proposition negated need both true and false q , so the proposition is a tautology.

1.1.14

$(\neg p \wedge (p \rightarrow q)) \rightarrow \neg q$ is a tautology, iff its negation has any solution:

$$(\neg p \wedge (p \rightarrow q)) \rightarrow \neg q$$

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| 1. | $\neg((\neg p \wedge (p \rightarrow q)) \rightarrow \neg q) \vee$ | Negation |
| 2. | $(\neg p \wedge (p \rightarrow q)) \wedge \neg \neg q \vee$ | Material nonimplication |
| 3. | $(\neg p \wedge (p \rightarrow q)) \wedge q \vee$ | Double negation elimination |
| 4. | $\neg p \wedge (p \rightarrow q) \wedge q \vee$ | Associative conjunction simplification |
| 5. | $\neg p \wedge (\neg p \vee q) \wedge q \vee$ | Material implication |
| | | |
| 6. | $\neg p \wedge \neg p \wedge q \vee \quad \neg p \wedge q \wedge q \vee$ | Disjunction split |
| 7. | $\neg p \wedge q \quad \quad \quad \neg p \wedge q$ | Associative conjunction simplification |

Since the proposition negated leads to a solution, the proposition is not a tautology.