Hand-in @ lecture #8

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1.5.24

> 'Identify the error or errors in this argument that supposedly shows that if $\forall x(P(x) \lor Q(x)$ is true then $\forall x P(x) \lor \forall x Q(x)$ is true.'

The argument

$\forall x(P(x) \lor Q(x))$	Premise	(1)
$P(c) \lor Q(c)$	Universal instantiation from (1)	(2)
$P(c)$	Simplification from (2)	(3)
$\forall x P(x)$	Universal generalization from (3)	(4)
$Q(c)$	Simplification from (2)	(5)
$\forall x Q(x)$	Universal generalization from (5)	(6)
$\forall x(P(x) \lor \forall x Q(x))$	Conjunction from (4) and (6)	(7)

is errornous at several places:

- Universal instantiation (2) leads to a particular c but is wrongly assumed arbitrary in universal generalizations (4 and 6) (from a particular c follows only *existential* generalization)
- A conjunction could be simplified to its components but the premise (1) is a disjunction
- Last transformation (7(is described as conjunction but the actually applied inference rule is disjunction.

1.6.16

> ‘Prove that if m and n are integers and mn is even, then m is even or n is even.’

The theorem states that $\forall m \forall n P((mn) \rightarrow P(m) \vee P(n))$, where $P(n)$ is ‘n is an even number’.

TODO

2.1.4

> ‘Suppose that $A = 2, 4, 6$, $B = 2, 6$, $C = 4, 6$, and $D = 4, 6, 8$. Determine which of these sets are subsets of which other of these sets.’

B and C are subsets of A, and C is a subset of D.

2.1.34a

> ‘Translate this quantifications into English and determine its truth value: $\exists x \in \mathbb{R}(x^3 = -1)$’

Some real number exists, that multiplied by itself thrice equals the number -1.

TODO

2.2.16a

> ‘Let A and B be sets. Show that $(A \cap B) \subseteq B$’

TODO